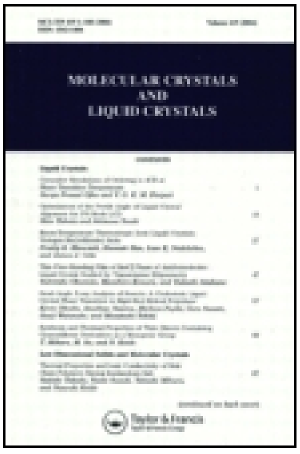

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DYNAMICS OF ECKHAUS MODES IN ONE-DIMENSIONAL ELECTROCONVECTION PATTERNS IN NEMATICS

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Abstract The quantitative experimental study of the Eckhaus instability in one-dimensional systems is carried out, employing electrohydrodynamic convection in a nematic layer. The obtained Busse balloon is in good agreement with that predicted by the Eckhaus theory. However, the observed changes of the wavenumber for unstable roll patterns, caused by the Eckhaus instability, as well as the growth rates for the corresponding spatial modes differ dramatically from those the theory yields for the most unstable Eckhaus modes. Possible reasons for the disagreement are discussed.

INTRODUCTION

The problem of pattern formation in open nonequilibrium systems, such as Rayleigh-Benard convection in isotropic fluids or electrohydrodynamic convection (EHC) in nematics, is a subject of intensive study for many years (for a collection of recent results see, e.g., Refs. 1-3). Nevertheless, even in the simplest case of roll pattern in one-dimensional (1D) systems at small value of the reduced control parameter, some important aspects of the problem remain unclear yet. In particular, since contrary to two-dimensional cases[1-5] the only instability steady 1D roll patterns may be subjected to is the Eckhaus instability, the latter has attracted considerable research activity both theoreticians[6-10] and experimentalists.[10-13] Presently theoretical understanding of the phenomenon is very detailed and profound. However, comparison of theoretical results with experiment remains uncomplete: it covers rather narrow ranges of variations of the pattern's wavenumber and is developed for a limited number of compared quantities. Besides, the experimental realizations of "one-dimensionality" of the systems under consideration were not quite perfect --- one of the best 1D approximation was achieved in Ref. 13, where the aspect ratio in y-direction (perpendicular to the pattern's wavenumber) was 3, so that two-dimensional distortions of roll patterns still could be substantial.

In the present research we try to fill in this gap between the theory and the experiment.

EXPERIMENTAL SETUP

To study the stability of 1D patterns we employed the standard frequency-voltage jump method in EHC in a nematic layer.[5,10,12-15] In order to obtain a perfect 1D system in EHC, we made on a glass plate a very narrow long transparent electrode (In_2O_3), which was lithographically etched as a strip 30 μm (±1μm) in width (y-dimension) and 1 cm in length (x-dimension), while x- and y-dimensions of another electrode both were about 1 cm. The electrodes were coated by a thin polymer film. The glass plates were assembled parallel to each other into a convective cell 50 μm in thickness. Thus, the aspect ratios of the cell were $\Gamma_x = 200$ and $\Gamma_y = 0.6$, respectively. The cell was filled with the nematic liquid crystal *p*-methoxybenzilidene-*p'*-*n*-butylaniline (MBBA). To obtain homogeneous planar alignments of the nematic, the surfaces of glass plates and the polymer film were rubbed in x-direction. The parallel and perpendicular conductivities of MBBA at the present experiment were 3.30 and $2.34 \times 10^{-9}\ \Omega^{-1}\mathrm{cm}^{-1}$, respectively (controlled by 0.012 wt% doping of TBAB). To avoid influence of sidewalls the convective patterns were observed about the center of the x-axis, so that the total area of observation was 14 % of the total length of the narrow electrode in x-direction. Images of convective patterns were taken by the CCD camera and recorded onto a magnetic tape as well as onto a magnetic disk. Then the records were analyzed by a personal computer. The above experimental setup was similar to that described in work,[15] which we refer to for more details.

The frequency-voltage jump method was employed in such a manner that the jump did not change the value of the reduced control parameter $\varepsilon = [V^2 - V_c^2(f_0)] / V_c^2(f_0)$, where V and f stand for the voltage and the frequency, respectively. The standard frequency f_0 in our experiment was 800 Hz, that corresponds to the critical voltage $V_c(f_0) = 13.95$ V and the critical wavenumber $q_c(f_0) = 14.15 \times 10^{-2}\ \mu\mathrm{m}^{-1}$. To obtain the roll pattern with a desired deviation of its wavenumber from the critical one $Q \equiv [q - q_c(f_0)] / q_c(f_0)$, the following technique was employed. Firstly, we created a normal roll pattern with a given wavenumber q, setting the frequency $f = f_1$, so that $q_c(f_1) = q$. Then the frequency was changed from f_1 to the standard frequency f_0, at the same time the applied voltage was assigned a new value, maintaining the value of ε fixed and equal to $[V^2 - V_c^2(f_1)] / V_c^2(f_1)$. The routine was employed repeatedly for a wide variety of values of Q and ε.

RESULTS AND DISCUSSIONS

The evolution of the initial roll pattern triggered by the jump, exhibit, depending on the concrete values of Q and ε, three qualitatively different kinds of dynamics: (a) --- spatially

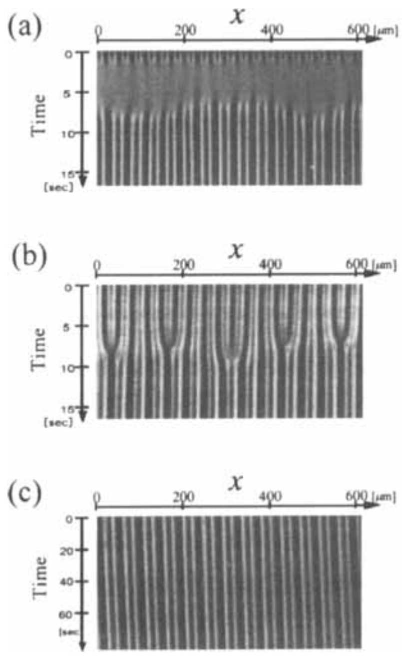

FIGURE 1 Spatiotemporal images of convection patterns for the frequency-voltage jump method. (a) The quiescent state is stable against the perturbation with given Q ($\varepsilon = 0.1, Q = 0.37$). (b) The Eckhaus instability ($\varepsilon = 0.3, Q = 0.37$). (c) The initial pattern is stable against the perturbation with given Q ($\varepsilon = 0.1, Q = 0.09$).

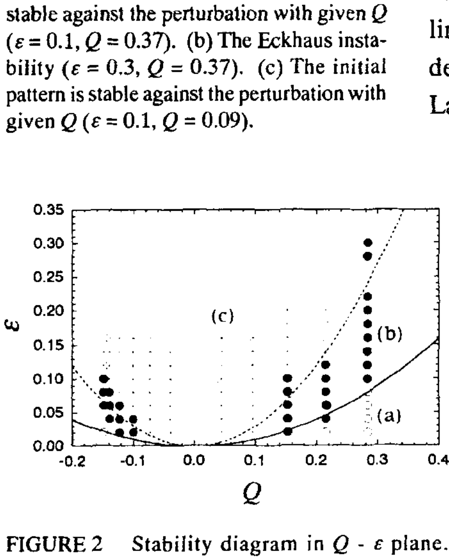

FIGURE 2 Stability diagram in Q - ε plane. Solid and dotted lines indicate respectively the neutral stability curve and the Eckhaus boundary (the Busse balloon) obtained from Eq. (1).

homogeneous replacement of the initial pattern by a new one --- the amplitude of the initial pattern gradually and spatially homogeneous vanishes and then, also gradually and spatially homogeneous, a new pattern with $q = q_c(f_0)$ appears [Fig. 1(a)]; (b) --- the replacement goes through annihilation or nucleation of pairs of rolls at spatially periodic positions [Fig 1(b)]; (c) --- the jumps does not result in any change of the spatial periodicity of the pattern [Fig. 1(c)]. We identify these three cases as follows: (a) --- the quiescent (convectionless) state is stable against perturbations with the given Q; (b) --- the given Q lies inside the band of the Eckhaus instability; and (c) --- the given Q corresponds to a stable pattern. Plotting these various results in the Q - ε plane we obtain the ***stability diagram***, shown in Fig. 2, where open and closed circles correspond to cases (a) and (b), and crosses to case (c).

The solid and the dotted lines in this figure indicate the neutral stability line and the Eckhaus line (the Busse balloon), respectively, which are derived from one-dimensional the Ginzburg-Landau (GL) equation:

$$\partial_T A = \partial_X^2 A + \left(1 - |A|^2\right) A. \qquad (1)$$

Here $A(X, T)$ is a complex amplitude, $X \propto \varepsilon^{1/2} x$ and $T \propto \varepsilon t$ are slow spatial and temporal variables, while x, t and ε correspond to the spatial coordinate, time and the reduced control parameter, respectively.[16] Thus, the wavenumber Q measured in the experiment is connected with the corresponding wavenumber of the GL equation κ by the relation $Q = const \cdot \varepsilon^{1/2} \kappa$, where the fitting constant is of order one.

It is well-known (see, e.g., Ref. 3, 7, 8) that the solution $A \equiv 0$ of Eq. (1) is stable against spatially periodic perturbations with $\kappa > 1$, that yields the neutral stability curve $\varepsilon = const{\cdot}Q^2$. At $\kappa < 1$ there is a family of steady spatially periodic solutions of Eq. (1), whose stability analysis against spatially periodic perturbations [$\propto \exp(\sigma T + ikX)$] yields the following dispersion relation for σ associated with the Eckhaus instability:[7,8]

$$\sigma = \kappa^2 - 1 - k^2 + \sqrt{\left(1-\kappa^2\right)^2 + 4\kappa^2 k^2}. \tag{2}$$

The growth rate $\sigma(\kappa, k)$ is positive (i.e., the solutions are unstable) if the condition

$$k^2 < 2\left(3\kappa^2 - 1\right) \tag{3}$$

holds. Inequality (3) cannot be satisfied at $\kappa^2 < 1/3$ that means stability of the solutions with these values of κ. The marginal value $\kappa = (1/3)^{1/2}$ yields the boundary of the Busse balloon that may be rewritten in the following form:

$$\varepsilon = 3 \cdot const \cdot Q^2, \tag{4}$$

with the same constant that enters into the expression for the neutral stability curve. Fitting the neutral stability curve to the experimental data, we obtain the value of *const*, that fixes the Busse balloon (3) shown in Fig. 2. Well agreement of Eq. (4) with the experimental results is clearly seen.

At $\kappa^2 > 1/3$ there is a band of unstable modes. Maximizing Eq. (2) at fixed $\kappa^2 > 1/3$, it is easy to obtain that the maximal growth rate for the unstable modes

$$\sigma_{max} = \frac{1}{4\kappa^2}\left(3\kappa^2 - 1\right)^2 \tag{5}$$

achieved at

$$k = k_{max} = \frac{1}{2\kappa}\sqrt{\left(3\kappa^2 - 1\right)\left(\kappa^2 + 1\right)}. \tag{6}$$

If the most unstable Eckhaus mode is the one that finally generates change of the pattern's wavenumber, k_{max} should be related to the long-wave modulation of the pattern, accompanying the change, and $1/\sigma_{max}$ to the characteristic induction time for nucleation or annihilation of pairs of rolls (τ_E). To check this assumption we tried to fit Eqs. (5), (6) to the quantities σ_E and K_E / q_c obtained in our experiment, where K_E was defined according to the relation $K_E = 2\pi / \lambda_E$ and λ_E denoted the average distance between two adjacent annihilation (nucleation) points.

Firstly, we tried to employ the conventional approach to the problem based on the GL equation written in the form:[16]

$$\tau \partial_T A = \xi^2 \partial_X^2 A + \left(1 - |A|^2\right) A \tag{7}$$

with two fitting parameters: the characteristic time τ and the correlation length ξ, choosing q_c as a third fitting constant. In this case there is the opportunity to obtain the fitting constants from independent measurements: namely, q_c from the position of the minimum of the neutral stability curve, ξ from the second derivative of this curve at $q = q_c$ and τ from temporal evolution of perturbations to the quiescent state. However, the resulting expressions for σ_{max} and k_{max} obtained according to this approach disagree with the experimental data for K_E and τ_E dramatically.

Better fit is achieved with four-parameter approximation of the form

$$\frac{K_E}{q_c} = p_K \cdot \frac{1}{2(aQ+b)} \sqrt{\left[3(aQ+b)^2 - \varepsilon\right]\left[(aQ+b)^2 + \varepsilon\right]}, \tag{8}$$

$$\tau_E^{-1} = p_\sigma \cdot \frac{1}{4(aQ+b)^2} \left[3(aQ+b)^2 - \varepsilon\right]^2, \tag{9}$$

where a, b, p_σ and p_K stand for fitting parameters. Fig. 3 displays comparison of the fitting functions (8), (9) with the experimental results. Despite a reasonable fit, we have to emphasize that the employed fitting functions are rather formal and do not have much physical meaning. Thus, the question of good description of the obtained experimental data for K_E and τ_E, actually, remains open.

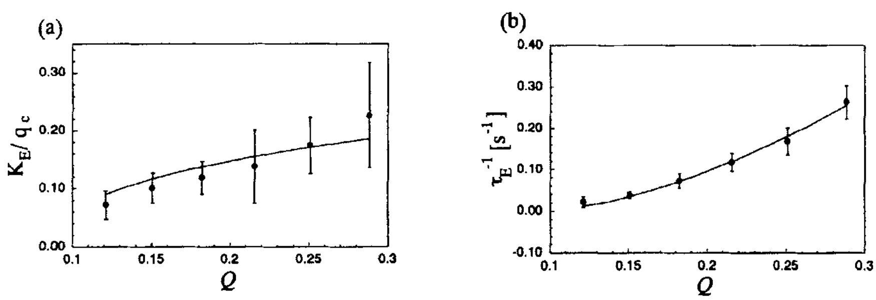

FIGURE 3 Q - dependence of the Eckhaus mode K_E (a) and its growth rate τ_E^{-1} (b) in our experiment. Dotted lines correspond to Eqs. (8) and (9) with the following values of fitting parameters (a least-squares fit): $a = 0.57$, $b = 0.13$, $p_K = 0.62 \pm 0.04$ and $p_\sigma = 3.23 \pm 0.08$ s^{-1}.

Note, that a certain deviation from k_{max} given by Eq. (6) for difference in the wavenumbers between initial unstable and the final stable roll patterns, was also observed in the computer simulation of the Eckhaus instability reported in Ref. 8. In this work the deviation was connected with phase diffusion process: Since different acts of nucleation (annihilations) of pairs of rolls in different points occur in different moments of time, the phase diffusion from the points where the acts already occurred and the local wavenumber of the pattern lies into the stable region to the regions where the local wavenumber still corresponds to an unstable pattern can stabilize these regions too and, hence, suppress further acts of nucleation (annihilations). The deviation may be caused by other alternative reasons also, e.g., it may be related to higher order nonlinear terms that are dropped in

Eq. (1) but that may be important at finite values of the control parameter, etc. Currently the detailed study of the problem is in progress and will be reported elsewhere.

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